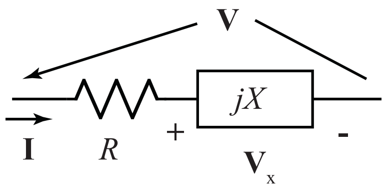
The Meaning of Q



For the impedance $Z = R + jX$, **V** is the (peak) phasor across the entire impedance, and $\mathbf{V}_x$ is the voltage across the reactive part. The complex power delivered to *Z* is:

$$\mathrm{S} = \frac{1}{2}\mathrm{VI}^* = \frac{1}{2}(Z\mathrm{I})\mathrm{I}^* = \frac{1}{2}|I|^2 Z = \frac{1}{2}|I|^2 R + j\frac{1}{2}|I|^2 X$$

Thus,

$$P = \mathrm{Re}(\mathbf{S}) = \frac{1}{2}I_m^2 R \quad \text{and} \quad Q = \mathrm{Im}(\mathbf{S}) = \frac{1}{2}I_m^2 X,$$

Where I_m is the peak value of the load current. We already know that *P* is the average power consumed by the load, so now we ask the question: what kind of power does *Q* represent?

Since *Q* is clearly associated with the reactive load, let's look at the instantaneous power $p_x(t)$ delivered to the reactive part of the load. Since $\mathbf{V}_x = jXI = jXI_m\angle\theta_I$, θ_I are the (peak) phase of the load current. This gives us,

$$v_x(t) = XI_m\cos(\omega t + \theta_I \pm 90°)$$

Where the + sign is for when $X > 0$ (inductive) and the - sign is for when $X < 0$ (capacitive) .

Hence, $p_x(t)$ is

$$p_x(t) = v_x(t)i(t) = XI_m^2\cos(\omega t + \theta_m)\cos(\omega t + \theta_m \pm 90°)$$
$$= \frac{XI_m^2}{2}\left[\cos(90°) + \cos(2\omega t + 2\theta_m \pm 180°)\right] = \frac{XI_m^2}{2}\cos(2\omega t + 2\theta_m \pm 180°)$$

Thus, the peak value of $p_x(t)$ is $\frac{XI_m^2}{2}$, which is the same as *Q*!

Hence, the quadrature power *Q* is the peak power delivered to (and then supplied by) the reactive part of the load.